Determination of aberrations by processing lenslet array image located on the CCD receiver

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ABSTRACT

The quality of large-sized astronomical optics is usually tested by the Hartmann technique. For the accuracy and operational efficiency checking of this techniques different schemes it is necessary to create the mathematical model. In this paper microlens array sensor simulation results and comparison of this scheme with other modifications of this technique is presented.

Algorithm, assisting to reach of successes in spots centers determination on the receiver, was created. This algorithm includes iterative the exact search and uses Fourier - image of the aperture of a lens. Research of the Shack-Hartmann testing scheme microlens array based, excluding Hartmann mask application is carried out. Modeling and optimization of the mesurement technique is provided, the real requirements to elements of the circuit are determined, mathematical apparatus and algorithm of processing of results of the given circuit is developed.

Keywords: Hartmann, Zernike, CCD receiver

1. INTRODUCTION

The work of large-sized optical devices occurs in complex temperature and seismic conditions, which influence the form of surfaces, answering for the image formation. For quality surveillance of a reflecting surface and elimination or indemnification of errors, well known approach, based on principles of the Hartmann technique, with application of computer processing of results, is widely used. This method allows to receive the complete information on errors of a mirror, which are represented by decomposition on Zernike polynomials. Essential lacks of a method from the point of view of data processing are low accuracy in spots recognition on CCD receiver and large volumes of calculations, influencing on general time of calculations.

For check of these and other parameters of system was developed mathematical model and software to increase spots coordinates measurement accuracy by the algorithm of iterative the exact search. The definition of hartmanogramm scale and orientation was made manually, that also complicated and reduced accuracy of results of the control. Besides such variant excluded an opportunity of direct monitoring of a mirror in an operating time.

Therefore at the analysis of hartmanogramm processing techniques were investigated and the ways of information reception about spot centers coordinates on hartmanogramm are advanced. It has allowed to simplify and to speed up process of reception of the data about transverse aberrations.

The processing of the testing results includes some stages:

1. Definition of ideal localizations of the spot centers - computing operation at known parameters of the standard.

2. Hartmanogramm processing - definition of the real spot centers.

2.1. Definition of orientation and scale of a snapshot.

2.2. Identification and separation of spots (definition of one spot area)

2.3. Definition of the spot centers - calculation of the energy spot center (the development of a technique of spot center definition) is necessary.

The stage of the hartmanogramm processing with the purpose of reception of exact coordinates of the energy spot centers is the basic component of definition process of beam transverse displacement from an ideal trajectory and, of wavefront aberration restoration.

2. HARTMANOGRAMM PROCESSING

The hartmanogramm registration now is making with the help of system for digital registration of the images on a photosensitive matrix of CCD.

The calculation of transverse aberrations, deformations of a mirror and reconstruction of a surface structure requires definition with high accuracy of points coordinates of crossing of rays with the image plane.

Separated territory of the image of the hartmanogramm one spot occupies on CCD some area. The definition of the localization of a maximum of distribution of intensity in the spread spot is a task of this stage.

For all schemes of the optics testing by the Hartmann technique this stage has no basic differences. After the separation of the one spot the stage of measurement of one spot follows.

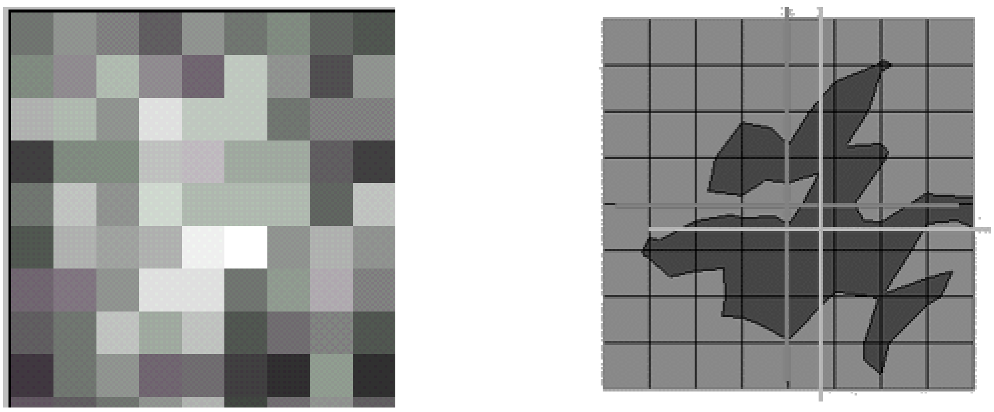
2.1. Method of definition of weight center of the chosen area

In this case the method of definition of the energy center of the spot on a weight center of the chosen area is traditional. The approached value of coordinates of the spot centers as weight centers (geometrical average) area allocated on hartmanogram.

For definition of the weight center of sample the standard expressions are used:

$$x_c = \frac{\sum m_i \cdot x_i}{\sum m_i}, \qquad y_c = \frac{\sum m_i \cdot y_i}{\sum m_i} \tag{1}$$

The researches have shown, that if the data have a casual error, that always takes place in real conditions, the method works much worse. The distribution of registered intensity within the limits of one spot is submitted in a fig. 1. a.



a) b)

Fig. 1. Distribution of intensity within the limits of one spot.
a) - model of a snapshot CCD, b) - topographical picture.

In a topographical picture the center of gravity (light line) and ideal center (dark line) fig. 1. b) is marked. The differences in this case make more than half of cell at noise 10 %.

From the carried out research it is clear, that the center of a stain is determined not precisely, and the additional processing, that is development of methods of the exact search is required.

3. THE EXACT SEARCH OF THE SPOT CENTER

3.1. Approximation with Gauss function

If the method is difractional limited, the Gauss function quite can correspond to distribution of intensity in a spot. Therefore for achievement of required accuracy sample of intensity on separated area is approximated as Gauss function through a method of the least squares, and then the coordinates of the spot center are determined as coordinates of the approximated function maximum.

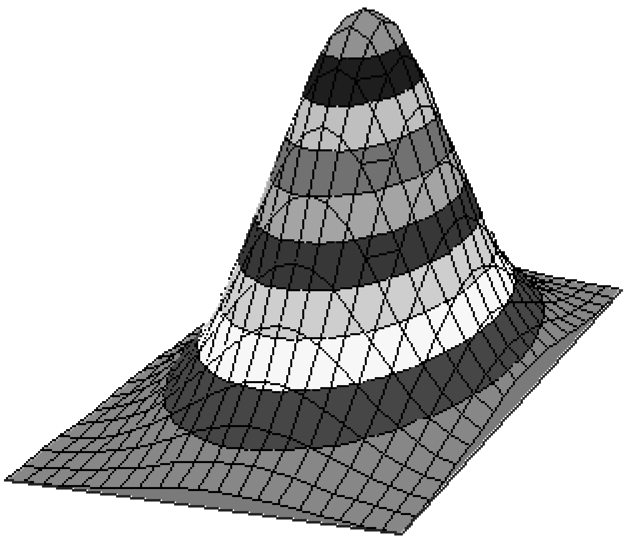


Fig. 2. A general view of the diagram of Gauss function

In the given work it is offered to investigate accuracy of approximation of intensity function distribution in a stain by Gauss function (Fig. 2) with the purpose of the exact search of the center. For approximation of the spot the parameters of function are specially selected, and the expression looks like:

$$f(x,y) = e^{-4\pi \frac{\left((x-x_c)^2+(y-y_c)^2\right)}{(d_{Airy})^2}}, \qquad (2)$$

where x_c, y_c - coordinate of the spot center, d_{Airy} - diameter of the Airy disk.

3.2. Developed method of the exact search of the spot center: approximation by a Fourier-image from the aperture

As distribution of intensity in the spot - diffractional picture, it is expedient to use for exacter approximation the Fourier transform, which square in optics is applied as mathematical diffractional model [1].

Proceeding from amount of points on CCD reseiver the parameters of sample get out. A step of points on CCD Δ_{CCD}.

$$K_{CCD} = \frac{d_{Airy}}{\Delta_{CCD}} = \frac{1{,}22 \cdot \lambda}{\Delta_{CCD} \cdot A'}, \tag{3}$$

And for a case of the square aperture:

$$K_{CCD} = \frac{\lambda}{\Delta_{CCD} \cdot A'}$$

K_{CCD} shows, how many points are stacked on a diameter of Airy disk.

The aperture target is known: $A' = \dfrac{D}{2 \cdot f'_{Lens}}$

Parameters for formation of a reference Fourier-image:

N – amount of points in reference pupil function is chosen from a number of degrees: $2^0, 2^1, 2^2 \ldots$ on the greatest similarity of a spot to a circle.

$$\frac{N}{NA} = K_{girth}, \; K_{CCD} = K_{girth}. \tag{4}$$

The diameter of the aperture is defined from the expression: $NA = \dfrac{N}{K_{girth}} = aperture\ diameter$ (5)

So-called pupil function (Fig. 3) for effect of area scanning reception by the aperture image is multiplied on factor of shift:

$$e^{2 \cdot \pi \cdot i \cdot \overline{\Delta X}} = \cos\left(2\pi \overline{\Delta X}\right) + i \cdot \left(2\pi \overline{\Delta X}\right) = f\left(\overline{\Delta X}\right) \tag{6}$$

The reception of the image with shift is described in such a way:

$$f_{pupil} \cdot f\left(\overline{\Delta X}\right) \cdot f\left(\overline{\Delta Y}\right) \xrightarrow{FFT} f_{image} \tag{7}$$

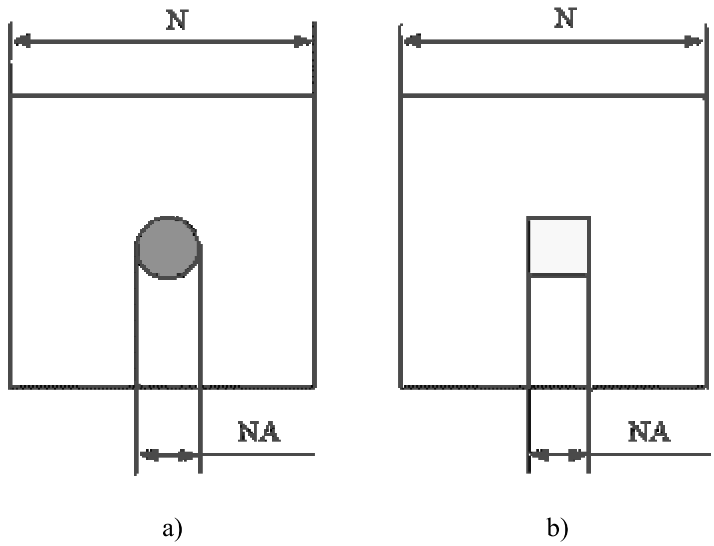


Fig. 3. The pupil function formation for approximation function reception of intensity dispersion in spot: a) - for the round aperture; b) - for the rectangular aperture.

Thus, account scanning multiplier for the account:

$$f\left(\overline{\Delta X}\right) = \exp\left(-2\pi i \cdot \frac{x_j \cdot a_x}{N}\right) \qquad (8)$$

Thus, knowing the size of the aperture, it is possible to receive a Fourier-image of the aperture and to scan by its square separated area.

3.3. The coordinate search algorithm developed for the exact search

At approximation both gauss function, and Fourier-image square, scanning all area of an arrangement of the spot is necessary. The execution time of this operation with the given accuracy is too great.

At modeling and optimization of the given process, the following attempts of reduction of time of scanning were undertaken:

1. The research of search of the weight center of sample has shown, that the mistake of definition of the center makes no more than one cells. Proceeding from this, it is possible to make scanning not of all sample, but only the areas consisting from 2x2 of pixels around of a found center of gravity.

2. The reduction of area of scanning considerably has reduced time of data processing, but the process of approximation has appeared still enough long. Therefore was developed and the algorithm of descent is applied.

Schematically the coordinate search process is shown in a Fig. 4.

Sequence of transition from approach to approach is following: for a beginning of descent the point of a weight center (1) is accepted, then the etalon function in four points around (2,3,4,5) is determined. The point with the least deviation from approximating function (5) gets out, and it is accepted for a new beginning of descent. The step is divided half-and-half.

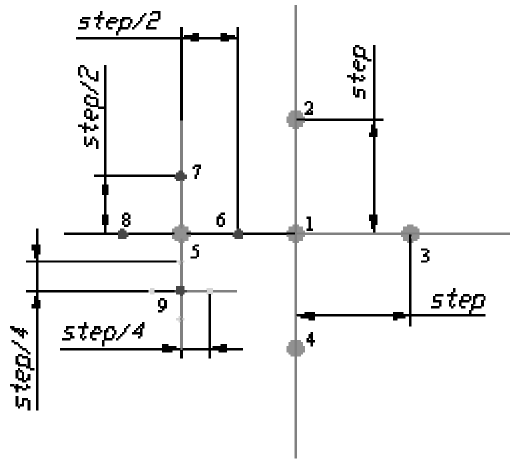
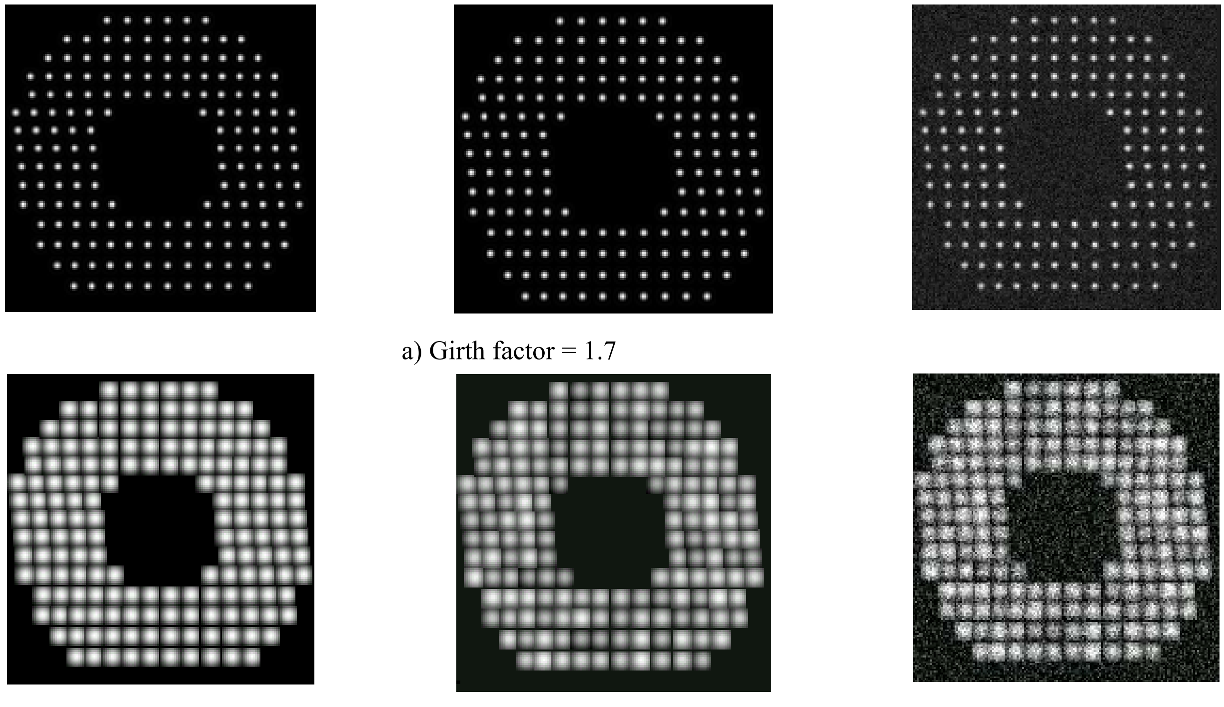


Fig. 4. The scheme of the approximating process

4. ANALYSIS OF THE ACCURACY OF THE DEVELOPED METHODS OF THE SPOT CENTER DEFINITION

4.1. The deformed initial data simulation

For testing the created mathematical model the formation of the deformed initial data is necessary. The analysis of the real hartmanogramm has shown, that the device of various kinds of distortions creation is necessary:



a) Girth factor = 1.7

b) Girth factor = 7

Fig. 5. A sequence of hartmanogramm creation with various values of girth factor.

For testing of work of methods the device of creation of various kinds of initial data distortion is developed.

Stages of modeling of the deformed initial data

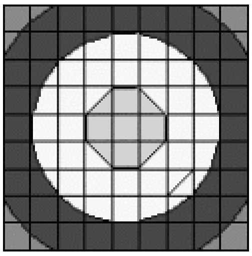
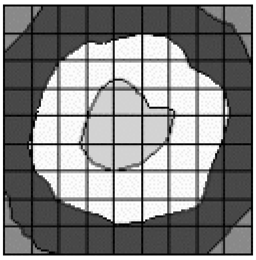
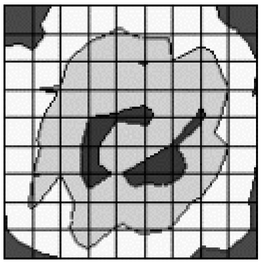
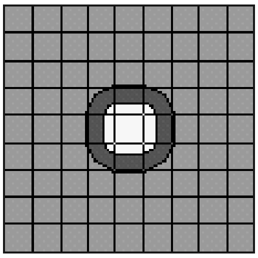
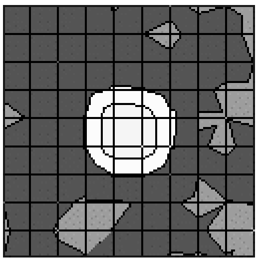
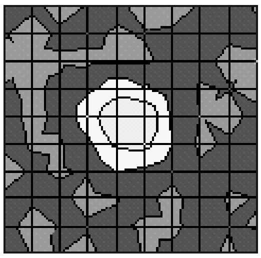
1. The Zernike coefficients, describing simulated deformation of the testing object are set.

2. The parameters of the testing scheme definition.
3. The transverse displacement of beam calculation.
4. The localization of the displaced spots definition.
5. The hartmanogramm picture, according to given aberrations formation.
6. Tne distortion of signal is made:
   6.1. On a field (reduction of the intensity to the edges of a pupil).
   6.2. On spots (non-uniform distribution of intensity in each spot).
   6.3. On structure of dispersion spot (uniform Gauss noise).

Thus, the picture of intensity distribution in a plane of the matrix receiver of the image as much as possible approached to real (Fig. 5) is simulated.

At enough of the initial data is received 7 points on the Airy disk diameter. The accuracy of definition of the center even in case of noise 10 %, on some orders is higher, than in a of the weight center method. The research of the exact search methods is given in the Table 1.

Table 1. The the exact search methods analysis

Data	Spot centers measurement accuracy		
	Without noice	Noice 3%	Noice 10%
Factor of girth = 7			
Weight center	10-15	10-3	10-1
The Gauss function scanning	10^{-15}	10^{-4}	10^{-2}
The square of Fourier image scanning	10^{-15}	10^{-5}	10^{-3}
Factor of girth = 2.5			
Weight center	10^{-15}	10^{-1}	1
The Gauss function scanning	10^{-15}	10^{-2}	10^{-1}
The square of Fourier image scanning	10^{-15}	10^{-4}	10^{-2}

The carried out research has shown high stability of the developed methods at various distortions of the initial data.

5. CONCLUSIONS

The analysis of the exact searchs methods (Table 1) shows significant improvement of the hartmanogramm processing stage. This advantage can be used on the next stages of the wavefront reconstruction [2].

6. REFERENCES

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