Gram-Schmidt technique for aberration analysis in telescope mirror testing

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ABSTRACT

For quality surveillance of large-sized astronomical optics in observatory conditions a Hartmann method is widely used. In the given article problems of wavefront and deformations restoration of a main mirror are examined by use of a Hartmann technique with small-sized mask in a converging beam for the telescope testing. Is shown, that by virtue of weak conditionality of restoration task the most suitable decision of a problem is Gram-Shmidt process, allowing at appropriate modification to receive best conditionality and consequently to minimize thus error of restoration.

Keywords: Hartmann, Zernike, Gram-Schmidt procedure, optical surface testing, high diameter telescopes.

1. INTRODUCTION

Modern optical astronomical devices are represented by complex engineering complexes. Thus in such devices means of self-checking and self-certification are provided.

The efficiency of a telescope is determined not only by diameter, which provides the spread pattern minimum size, but also lot of other factors - sensitivity of the receiver of radiation, accuracy of system of tracking of a telescope, background light of the night sky, astroclimate in observatory region and microclimate in a telescope tower. But the more important factor is residual deformations of a mirror surface aberrating wavefront and essentially lowering the image quality.

In this paper a technique of mathematical processing the results received after the identification of spots, on the hartmanogramm, in order to restore the deformation of the surface being tested.

2. THE ASTRONOMICAL OPTICS TESTING BY MEANS OF THE HARTMANN TECHNIQUE

The basic principle of methods of the control with the mask application is that the wave front is estimated in a number of previously chosen points and the results are compared to theoretical values[2]. The wave front is estimated by several beams, perpendicular to it, and the deviation of light traces from their ideal rules is fixed on some recording plane
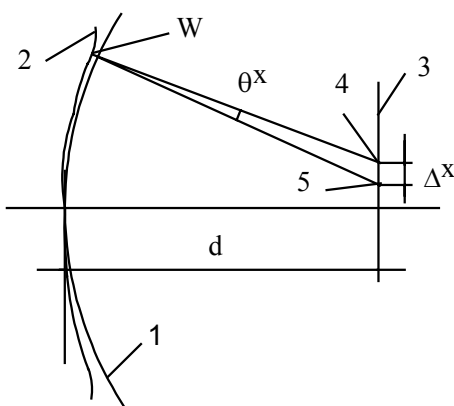

Fig. 1. The raytrace of beams in the Hartmann method:
1 - Ideal wave front; 2 - real wave front; 3 - the registration plane;
4 - real position of a beam; 5 - ideal position of a beam.

On the drawing are shown:

Δx - Meaning of one part of beam deviation; d - distance between a mirror and plane of supervision; θx - angular tilt.

The resulting deviation from the ideal surface form h, is connected with a deviation of wavefront W by the ratio $W = 2 \cdot h$.

The relation between wavefront aberration W and part x of beam deviation in supervision plane can be received from the following expression:

$$\frac{\partial W}{\partial x} = \frac{\Delta x}{d}$$

(1)

After integration, and translation of the surface deviation h in wavelenghts, we shall receive:

$$H = \frac{1}{2d\lambda} \int_0^x \Delta x \, dx$$

(2)

As the function Δx is determined only in fixed points - analogues of apertures of the mask, the integration will be usually carried out by the formula of a trapeze:

$$H_N = \frac{1}{2d\lambda} \cdot \sum_{n=2}^{N} \left(\frac{\Delta x_{n-1} + \Delta x_{n+1}}{2} \right) \cdot \delta x_{n-1}$$

$$H_M = \frac{1}{2d\lambda} \cdot \sum_{m=2}^{M} \left(\frac{\Delta y_{m-1} + \Delta y_{m+1}}{2} \right) \cdot \delta y_{m-1}$$

(3)

Other integration methods result in more reliable results.

Then it is possible to describe the wavefront in terms of Zernike polynomials. Theoretically this method is ideal, because in it there is not enough error of methods of numerical integration[2]. The difficulty consists in a choice of suitable polynomial, because such approach can bring unreasonable smoothing of sharp ledges having place or to introduction of any changes, absent actually. It occurs by use polynomials of the too small or too high orders. There is no necessity to speak that in result reliability of results of the control especially near the mirror edge is reduced.

Obviously one of main difficulties in methods of the control with the application of the mask consists in occurrence of errors, connected with transformation of the surface deviation data. The errors can be reduced, if a control technique allows reception of the necessary size by several independent ways, as at the presence of an error the most reliable result is reached at repeated recurrences and subsequent smoothing of testing data.

Thus the method of the control with mask application allowing to use several independent ways of reception of a surface deviation in chosen points can give the best approximation to the valid meaning of a deviation than method not using this procedure.
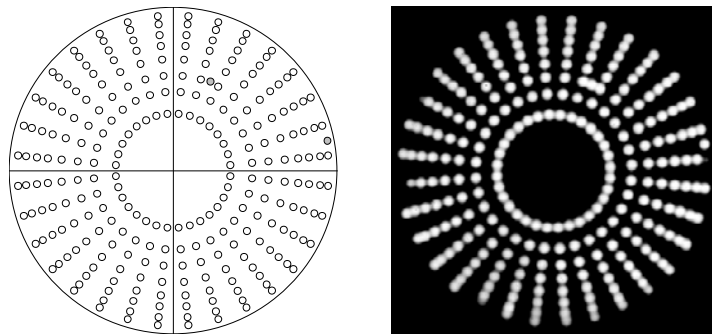

Fig. 2. The scheme of an arrangement of apertures on the radial Hartmann mask and its appropriate hartmanogramm picture.

The basic principle of the control with the help of radial masks is that on concave specular surface with the circular aperture we choose a number of points with the help of small apertures in the opaque screen located on direct along several diameters of a mirror (Fig. 2). The picture of such type has number of advantages: the analysis of the round aperture in polar coordinates is facilitated and opportunity to reveal the most common defects of surfaces - zone errors, concentric "hills" and "holes" a few share of micrometer size, being the result of the classical methods of grinding and lapping. Thus the radial screen provides simplicity of the analysis of the testing results of the basic kinds of superficial defects.

2.1. TELESCOPE MAIN MIRROR CONTROL WITH SMALL-SIZED HARTMANN MASK IN A CONVERGENT BEAM

For the astronomical telescopr control Hartmann method with full-sized mask, established in a collimated beam before objective is usually used. However, on large diameters of a mirror the application of such mask encounters significant difficulties, connected to its installation before the control and removal after.

For the control of a Big Alt-Azimutal Telescope (BTA) main mirror was used small-sized Hartmann mask, established in a convergent beam near the focus of a main mirror [3,4,5].

Structurally it was carried out as apertures in a cover of a glass of main focus and at any moment it could be easily entered into a course of beams for the control and then as is easily removed.
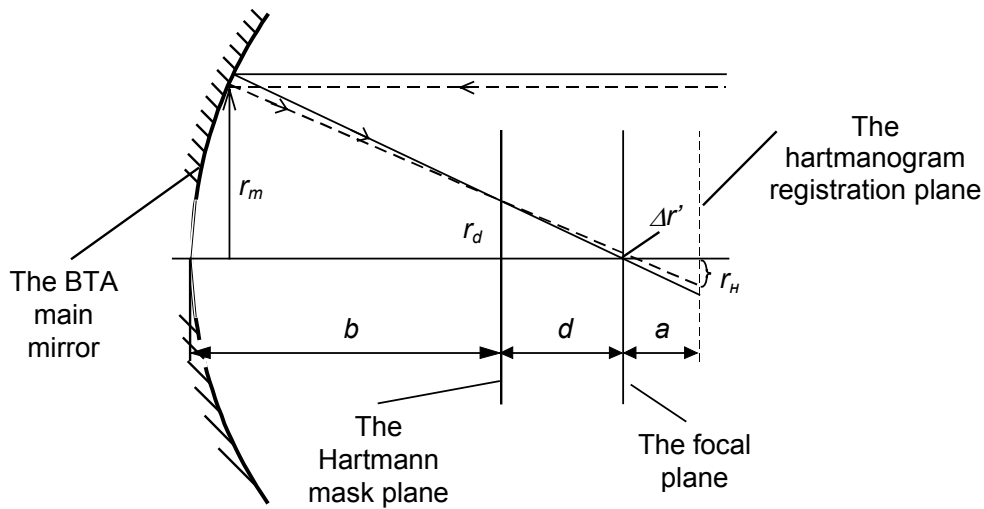

Fig. 3. The scheme of the BTA main mirror testing with small-sized Hartmann mask in a convergent beam

On the Fig. 3 beams, going through the center of any aperture in Hartmann mask, are displayed. The continuous line shows a beam, appropriate to an ideal mirror, dotted line - deformed.

The beam coordinates in the Hartmann mask plane are identical to ideal and indignant beams and make vector r_d of coordinates (x_d, y_d). In the focal plane the indignant beam has coordinates $(\Delta x, \Delta y)$, forming a transverse aberration vector Δr. This ray crosses the hartmanogramm registration plane forms a spot, the centre of which has coordinates (x_H, y_H), forming a vector r_H. At last, on the mirror surface the indignant beam has coordinates (x_m, y_m) of trial point, forming a vector r_m.

Easily to establish relation between these coordinates, counting known vector r_d and vector r_H:

$$x_H = \Delta x \cdot \left(1 + \frac{a}{d}\right) - x_d \cdot \frac{a}{d} + c_x , \quad y_H = \Delta y \cdot \left(1 + \frac{a}{d}\right) - y_d \cdot \frac{a}{d} + c_y \tag{4}$$

where c_x, c_y - transverse displacement of a matrix receiver center, beforehand unknown.

Our purpose is to calculate the transverse aberration, however previously from expressions (4) it is necessary to specify not precisely known distance a and unknown displacement c_x, c_y.

This problem can be solved by application the least squares method, minimizing a sum of squares transverse aberrations, then specified transverse aberrations $\Delta x, \Delta y$ are obtained from expression (5).

As is known, transverse aberrations there are private derivatives from wave aberraions on initial (relative) pupil coordinates:

$$\Delta x = \frac{\partial W}{\partial \rho_x} / A', \Delta y = \frac{\partial W}{\partial \rho_y} / A' \tag{5}$$

Where W - wave aberration, expressed in an absolute measure (in this task in mm), ρ_x, ρ_y - initial (relative) pupil coordinate of a beam on a mirror, determined by expressions:,

$$\rho_x = \frac{2 \cdot x_m}{D}, \quad \rho_y = \frac{2 \cdot y_m}{D}, \tag{6}$$

A' - Back aperture of a mirror.

$$A' = \frac{D}{2f'}, \tag{7}$$

where x_m, y_m - coordinate of a beam (trial point) on a mirror, D - diameter of a mirror, f' - it's focal distance.

The coordinates (x_m, y_m) can be found with hlp of known transverse aberrations and coordinates of an aperture in Hartmann mask (x_d, y_d) from the equation of a meeting of a beam with rotation paraboloid, what is a mirror.

Thus, we from the expressions (5) have transverse aberrations, from the equations of a meeting and from the formulas (6) and (7) - initial coordinates of trial points on a mirror.

3.THE PROCESSING OF THE HARTMANN TEST RESULTS

The calculation of transverse aberrations, deformations of a mirror and reflection surface structure construction requires definition with high accuracy of light beams crossing points coordinates, passed through Hartmann mask, with a plane of the image. Until the last time hartmanogramms were registered on photographic plates, the analysis was made manually, and the measurements were carried out at visual focusing on spot center on two-coordinate measuring devices with accuracy of measurement of spot centers $\pm 10 \div 12$ micron. With the purpose of accuracy increasing up to $\pm 3 \div 5$ microns the several hartmanogramm analysis and averaging of received results was made.

Therefore it is necessary to automate the analysis and to receive values of spot centers coordinates on hartmanogramm with accuracy of about $\pm 3 \div 5$ microns.

3.1. HARTMANOGRAMM SCALE AND ORIENTATION DEFINITION. SPOT IDENTIFICATION.

The hartmanogramm registration is made with the help of a system for digital registration of the images on a CCD matrix. The system for digital registration of the images allows to keep the image as binary files with textual heading (FITS-format).

For the BTA primary mirror control the classical radial screen Hartmann with 8 radial lines, 32 points in each (Fig. 2) is used. Hartmann mask parameters, so and the coordinates of all its apertures, are known and will be used for transverse aberrations definition, calculated as a difference between spots coordinates on real and ideal hartmanogramm.

The preliminary hartmanogramm analysis consists in of all sample intensity values gravity centre determination on hartmanogramm. After the centre of gravity is chosen as a beginning of coordinate system, with the help two reper points in the diagram top right part it is necessary to define hartmanogramm scale and orientation.

On known Hartmann mask parameters now it is easy to define scale M of the hartmanogramm real centre and it's displacement from ideal. For coordinates transformation now it is necessary to use the following formulas:

$$x' = mx + x_c; \ y' = my + y_c; \tag{8}$$

Besides reper points position gives the hartmanogramm orientation unequivocally. The identification of spots carry out under the accepted order of apertures numbering on mask, that is conformity of spots to certain apertures is made. Thus, determining the order of mask apertures detour.

Approached values of spots centers coordinates as of gravity centres (geometric average) area allocated on hartmanogramm is obtained at first. Then for achievement of required accuracy sample of intensity on separated area aproximated as Haussoide my means of the least squares method, and then the coordinates of spot centre are defined as coordinates of approximated function maximum. Development of a method with the help of use Haussoide of the higher order is hereinafter supposed.

3.2. THE INFORMATION ABOUT MIRROR SURFACE DEFORMATION.

As wave aberration is connected to the mirror deformation by coefficient -2, from the formula (9) we can receive expressions for private derivative mirror deformation on initial coordinates in trial points.

$$\Delta W = -2 \cdot \Delta Z \tag{9}$$

ãäå ΔW - wave front deformation, formed by a mirror, à ΔZ - required deformation of a mirror surface.

This is the initial equations for deformation definition, restoring it on partial derivatives in trial points.

4. MATHEMATICAL APPARATUS TELESCOPE MAIN MIRROR DEFORMATION RESTORATION

In astronomical systems the large importance has decomposition on Zernike polynomials, orthogonal on a annular area with given factor of central obsquration ε .

For the approximation of wave aberration function the most reasonable will be the use of basis, orthogonal on ring area.

4.1. CONSTRUCTION OF APPROXIMATION BASIS

The orthogonal Zernike polynomials dependence from polar coordinates in general is:

$$P_n^{\pm m}(\rho \cdot \cos\varphi, \rho \cdot \sin\varphi) = R_n^m(\rho) \exp(\pm i \cdot m \cdot \varphi) , \tag{10}$$

where m, n - integer positive numbers, have restriction as: $n \geq m$, moreover the difference $(n - m)$ should be even number.

The most effective way of Zernike polynomials values calculation consists in use Forsythe three-member expressions:

$$Q_{k+1}^m(t) = \gamma_{k+1}^m \left[\left(t - \alpha_{k+1}^m \right) \cdot Q_k^m(t) - \beta_{k+1}^m \cdot Q_{k-1}^m(t) \right] \tag{11}$$

Factors of these expressions $\alpha_k^m, \beta_k^m, \gamma_k^m$, are calculated proceeding from the requirements of orthogonality area form (circle or ring). The parameters of a ring are determined by obsquration. Integrals for calculation of factors α_k^m and norm λ_k^m are obtained numerically.

Directly for approximation transverse aberrations not only polynomials calculation is required, but also their private derivatives on initial coordinates, as:

$$\Delta\eta_x = \frac{\partial W}{\partial \rho_x}, \quad \Delta\eta_y = \frac{\partial W}{\partial \rho_y} \tag{12}$$

Private derivative orthogonal on a ring Zernike polynomials on initial coordinates are calculated with use of a Forsythe expression, with coefficients, calculated for orthogonality area:

$$Q'^{m}_{k+1}(t) = \gamma^{m}_{k+1}\left[\left(t - \alpha^{m}_{k+1}\right)\cdot Q'^{m}_{k}(t) - \beta^{m}_{k+1}\cdot Q'^{m}_{k-1}(t) + Q^{m}_{k}(t)\right] \tag{13}$$

With use of mentioned expressions we have an opportunity to receive values as Zernike polynomials, and their private derivatives, orthogonal on a ring and on a circle.

4.2. THE CONSTRUCTIONAL MATRIX FORMATION.

In the given part of submitted work initial data for determination of wave aberration approximation coefficients on Zernike polynomials, (orthogonal on ring area with central obsquration ε), caused deformation of a mirror are transverse aberrations values.

The decomposition of wave aberration on orthogonal on Zernike polynomials $W(\rho_x, \rho_y)$ in polar coordinates looks like:

$$W(\rho, \varphi) = \sum_{n=0}^{p}\sum_{m=0}^{n}c_{nm}R_n^m(\rho)\cos(m\varphi) + \sum_{n=1}^{p}\sum_{m=1}^{n}s_{nm}R_n^m(\rho)\sin(m\varphi), \tag{14}$$

Where c_{nm} and s_{nm} - factors of decomposition, which serve common numerical representation of optical system aberration. They are subject to calculation.

The expression for transverse aberrations calculation looks like:

$$\Delta x' = \frac{-\lambda}{A_x'}\cdot\frac{\partial W}{\partial \rho_x'}, \quad \Delta y' = \frac{-\lambda}{A_y'}\cdot\frac{\partial W}{\partial \rho_y'} \tag{15}$$

From the expression (15) it is clear, that for calculation of wave aberration decomposition coefficients on Zernike polynomials the following data are necessary:

- collection of transverse aberrations values $\Delta x'$ and $\Delta y'$ in various points of pupil, received after test results processing,
- testing points coordinates,
- collection of Zernike polynomials derivatives values, the testing points calculated for.

For decomposition factors determination it is necessary to present initial data in such a way, defined by a calculation technique used in this work.

If to present equations system in the matrix form, the approximation coefficients will be the decision of linear equations system, looks like:

$$\mathbf{A}\cdot\mathbf{C} = \mathbf{B} \tag{16}$$

where $\mathbf{A}$ - task constructional matrix. The matrix structure is shown below:

$$
\mathbf{A} = \begin{pmatrix}
1 & \dfrac{\partial\left(R_2^0(\rho)\right)}{\partial\rho_x}\bigg|_{(\rho_{x1},\rho_{y1})} & \cdots & \dfrac{\partial\left(R_n^m(\rho)\cdot\cos(m\cdot\varphi)\right)}{\partial\rho_x}\bigg|_{(\rho_{x1},\rho_{y1})} & \cdots \\[2em]
1 & \dfrac{\partial\left(R_2^0(\rho)\right)}{\partial\rho_y}\bigg|_{(\rho_{x1},\rho_{y1})} & \cdots & \dfrac{\partial\left(R_n^m(\rho)\cdot\cos(m\cdot\varphi)\right)}{\partial\rho_y}\bigg|_{(\rho_{x1},\rho_{y1})} & \cdots \\[2em]
\cdots & \cdots & \cdots & \cdots & \cdots \\[1em]
1 & \dfrac{\partial\left(R_2^0(\rho)\right)}{\partial\rho_x}\bigg|_{(\rho_{xN},\rho_{yN})} & \cdots & \dfrac{\partial\left(R_n^m(\rho)\cdot\cos(m\cdot\varphi)\right)}{\partial\rho_\times}\bigg|_{(\rho_{xN},\rho_{yN})} & \cdots \\[2em]
1 & \dfrac{\partial\left(R_2^0(\rho)\right)}{\partial\rho_y}\bigg|_{(\rho_{xN},\rho_{yN})} & \cdots & \dfrac{\partial\left(R_n^m(\rho)\cdot\cos(m\cdot\varphi)\right)}{\partial\rho_y}\bigg|_{(\rho_{xN},\rho_{yN})} & \cdots
\end{pmatrix}
$$

The rows amount in matrix is equal to amount of initial data, that is double amount of points. Each pair of rows corresponds to the transverse aberration on x and on y accordingly. The columns amount in matrix is equal to amount of coefficients, used for wave aberration approximation. The coefficients and transverse aberrations are submitted as vectors

$\mathbf{C}$ - aberration coefficients vector

$$
\mathbf{C} = \begin{pmatrix} c_{20} \\ c_{40} \\ \cdots \\ c_{nm} \\ s_{11} \\ \cdots \\ s_{nm} \end{pmatrix}
$$

$\mathbf{B}$ - transverse aberrations values vector, it's structure corresponds to $\mathbf{P}$ matrix:

$$
\mathbf{B} = \begin{pmatrix} \Delta\eta_{x1} \\ \Delta\eta_{y1} \\ \cdots \\ \Delta\eta_{xN} \\ \Delta\eta_{yN} \end{pmatrix}
$$

4.3. WAVE ABERRATION APPROXIMATION COEFFICIENTS CALCULATION BY MEANS THE LEAST-SQUARES METHOD.

Accordingly to the least squares method the equations system (16) should be transformed in such a way:

$$
\left(\mathbf{A}^\mathbf{T}\cdot\mathbf{A}\right)\cdot\mathbf{C} = \mathbf{A}^\mathbf{T}\cdot\mathbf{B} \tag{17}
$$

and this expression can be written down as:

$$
\mathbf{D}\cdot\mathbf{C} = \mathbf{F} \tag{18}
$$

The system decision in a general kind will look as this:

$$
\mathbf{C} = \left(\mathbf{A}^\mathbf{T}\cdot\mathbf{A}\right)^{-1}\mathbf{A}^\mathbf{T}\cdot\mathbf{B} \tag{19}
$$

or:

$$
\mathbf{C} = \mathbf{D}^{-1}\cdot\mathbf{F} \tag{20}
$$

For this decision realization we shall use a Gram-Schmidt orthogonalization procedure, with help of what we shall receive values of coefficients $\mathbf{C}$.

Applying this method, we orthogonalize matrix rows and shall transform that to an orthogonal matrix $\mathbf{U}$, which has property:

$$
\mathbf{U}^{-1} = \mathbf{U}^\mathbf{T} \tag{21}
$$

Simultaneously with matrix orthogonalization procedure there are the similar transformations of with vector of right parts $\mathbf{B}$, therefore instead of $\mathbf{H}$ we have $\mathbf{H}^{'}$.

Then we receive new system of linear equations:

$$\mathbf{U} \cdot \mathbf{C} = \mathbf{H}^{'} \tag{22}$$

In orthogonalization process, carried out on optimum strategy, optimum choice of rows in matrix $\mathbf{D}$, ensuring minimum conditionality, set as the relative admission on line norm size is made.

Besides an opportunity to make the regression analysis of a norms vector by one of statistical criteria (Fisher's criterion, criterion χ^2 and etc.) is available.

After processing the final matrix $\mathbf{U}$ can be used for the task solving, using orthgonatily property:

$$\mathbf{C} = \mathbf{U}^{\mathbf{T}} \cdot \mathbf{H}^{'} \tag{23}$$

The wave aberration decomposition on Zernike polynomials coefficients calculation by means of the least squares method consists of the following stages:

Initial data: a matrix of the left parts $\mathbf{A}$, vector of the right parts $\mathbf{B}$, vector of decision result of system of equations $\mathbf{C}$.

- Transposition of a matrix of the left parts $\mathbf{A}$.

- Calculation of multiplication for the right part $\mathbf{H} = \mathbf{A}^{\mathbf{T}} \cdot \mathbf{B}$:

- Calculation of multiplication $\mathbf{D} = \left(\mathbf{A}^{\mathbf{T}} \cdot \mathbf{A} \right)$ for a constructional matrix of a problem.

- The matrix orthogonalization $\mathbf{U} = \mathbf{D}^{-1}$

- The orthogonal matrix transposition $\mathbf{U}^{\mathbf{T}}$

- The right parts matrix transformation for conformity with left, reception of a matrix $\mathbf{H}^{'}$.

- Solution of the equation $\mathbf{C} = \mathbf{U}^{\mathbf{T}} \cdot \mathbf{H}^{'}$

5. THE GRAM-SHMIDT METHOD REALIZATION.

For result achievement and wave aberrations approximation coefficients calculations, it is nesessary to calculate matrix, inverse to $\mathbf{D} = \left(\mathbf{A}^{\mathbf{T}} \cdot \mathbf{A} \right)$, see (19), (20).

Lets consider matrix $\mathbf{D}$ orthogonalization process and its realization with by means of Gram-Shmidt method, being based on its formulation in work [1]:

Let is given accounting (final or infinite) set of linearly independent functions $\varphi_1(x), \varphi_2(x), \ldots$, normalized on some area, then there is orthonormalized sequence of functions $u_1(x), u_2(x), \ldots$, inducing the same variety of functions. This sequence can be constructed by means of following formulas, which carry out Gram-Shmidt orthogonalization process.

$$\left. \begin{aligned} u_i(x) &= \frac{v_i(x)}{\|v_i(x)\|} = \frac{v_i(x)}{\sqrt{(v_i, v_i)}}, \qquad (i = 1,\ 2, \ldots) \\ v_1(x) &= \varphi_1(x), \quad v_{i+1}(x) = \varphi_{i+1}(x) - \sum_{k=0}^{i} (u_k, \varphi_{i+1}) \cdot u_k(x) \end{aligned} \right\} \tag{24}$$

As show expressions (24), in this case considered matrix columns (vectors) orthogonalization. In our case it is for some reasons required to apply rows orthogonalization, instead of columns of the constructional matrix of a problem.

5.1. FORMATION OF AN ORTHOGONAL MATRIX U AND AUXILIARY MATRIX V.

1. The first line corresponds to the first line of an initial matrix $\mathbf{D}$.

$$v_{00} = d_{00}, \ v_{01} = d_{01}, \ v_{02} = d_{02}, \ v_{03} = d_{03}, \ \ldots, v_{0j} = d_{0j}.$$

$$\mathbf{V}_0 = \mathbf{D}_0$$

Norm of the matrix $\mathbf{V}$ first line:

$$\|\mathbf{V}_0\| = (\mathbf{V}_0 \cdot \mathbf{V}_0) = d_{00} \cdot d_{00} + d_{01} \cdot d_{01} + d_{02} \cdot d_{02} + d_{03} \cdot d_{03} + \ldots$$

According to the formula: $\mathbf{U}_i(x) = \dfrac{\mathbf{V}_i(x)}{\|\mathbf{V}_0\|}$, the line of a matrix $\mathbf{U}$ looks like:

$$\mathbf{U}_0 = \frac{\mathbf{D}_0}{\|\mathbf{V}_0\|}$$

2. After that it is nesessary to make account of scalar multiplication of initial matrix current line and all lines calculated on previous steps of an orthogonal matrix:

$$(\mathbf{U}_k \cdot \mathbf{D}_i) = u_{k0} \cdot d_{i0} + u_{k1} \cdot d_{i1} + u_{k2} \cdot d_{i2} + u_{k3} \cdot d_{i3} + \ldots.$$

3. A current line of a matrix $\mathbf{D}$ we deduct the previous lines of a matrix $\mathbf{U}$, multiplied by the scalar products appropriate to them

$$v_{10} = d_{10} - u_{00} \cdot (\mathbf{U_0} \cdot \mathbf{D_1}), \ v_{11} = d_{11} - u_{01} \cdot (\mathbf{U_0} \cdot \mathbf{D_1}), \ v_{12} = d_{12} - u_{02} \cdot (\mathbf{U_0} \cdot \mathbf{D_1}),$$
$$\ldots, v_{1j} = d_{1j} - u_{0j} \cdot (\mathbf{U_0} \cdot \mathbf{D_1}).$$

Thus for matrix $\mathbf{V}$:

$$\mathbf{V}_1 = \mathbf{D}_1 - (\mathbf{U}_0 \cdot \mathbf{D}_1) \cdot \mathbf{U}_0.$$

For an orthogonal matrix :

$$\mathbf{U}_1 = \frac{\mathbf{D}_1 - (\mathbf{U}_0 \cdot \mathbf{D}_1) \cdot \mathbf{U}_0}{\|\mathbf{V}_1\|}.$$

4. Thus, logically on orthogonalization steps for the auxiliary matrix we have:

$$\mathbf{V}_i = \mathbf{A}_i - \sum_{k=0}^{i-1} (\mathbf{U}_k \cdot \mathbf{A}_i) \cdot \mathbf{U}_k \tag{25}$$

and, accordingly, for an orthogonal matrix:

$$\mathbf{U}_i = \frac{\mathbf{A}_i - \sum_{k=0}^{i-1}\left(\mathbf{U}_k \cdot \mathbf{A}_i\right)\cdot \mathbf{U}_k}{\|\mathbf{V}_i\|} \tag{26}$$

5.2. TRANSFORMATION OF THE RIGHT PARTS VECTOR.

Obviously, that such modifications of a constructional matrix should bring to the appropriate transformations of a right parts vector, so that the initial equations remained correct.

Otrhogonalization process influences the left part of linear equations system (22). To illustrate otrhogonalization process and actions accompanying it, we shall consider, what way it is necessary to transform a right parts matrix for preservation conformity of a changed equation left part $\mathbf{H}$, by analogy with orthogonalized $\mathbf{D}$.

Intermediate data, received in otrhogonalization process, are used for transformation. The algorithm of transformation process is similar by what is used for matrix otrhogonalization by means of Gram-Shmidt method.

Lets denote current line norm $\|\mathbf{V}_i\|$ of auxiliary matrix $\mathbf{V}$ as $\mathbf{S}_i$. Then:

Matrix $\mathbf{D}$ otrhogonalization	Vector $\mathbf{H}$ transformation appropriate to the otrhogonalization process
$\mathbf{U}_0 = \dfrac{\mathbf{D}_0}{\mathbf{S}_0}$, $\mathbf{U}_1 = \dfrac{\mathbf{D}_1 - \left(\mathbf{U}_0\cdot\mathbf{D}_1\right)\cdot\mathbf{U}_0}{\mathbf{S}_1}$	$\mathbf{H}_0' = \dfrac{\mathbf{H}_0}{\mathbf{D}_0}$, $\mathbf{H}_1' = \dfrac{\mathbf{H}_1 - \left(\mathbf{U}_0\cdot\mathbf{D}_1\right)\cdot\mathbf{H}_0'}{\mathbf{S}_1}$
The general expression for all lines of matrixes $\mathbf{D}$ and $\mathbf{H}$ looks like:	
$\mathbf{U}_i = \dfrac{\mathbf{D}_i - \sum_{k=0}^{i-1}\left(\mathbf{U}_k\cdot\mathbf{D}_i\right)\cdot\mathbf{U}_k}{\mathbf{S}_i}$	$\mathbf{H}_i' = \dfrac{\mathbf{H}_i - \sum_{k=0}^{i-1}\left(\mathbf{U}_k\cdot\mathbf{D}_i\right)\cdot\mathbf{H}_k'}{\mathbf{S}_i}$

Thus, after realization of considered procedures, components $\mathbf{U}$ and $\mathbf{B}'$ of the transformed linear equations system (22), are obtained. The decision of this system will be calculation according to the expression (23).

6. METHOD ACCURACY CONSIDERATION

The created model was tested as follows: on given Zernike coefficients, calculation of transverse aberrations was made, values of what was noised. The results of approximation not strongly differ from initial data. On calculated coefficients it is easy to construct a topographical plot of deformations of a mirror. On Fig. 4. topographical plot of deformations of a mirror surface for several aberration types and for random aberration coefficients are presented.

For comparison results of modeling with and without obsquration are submitted. The last drawings are received with random values of Zernike coefficients.

Coefficients	Without obsquration	With obsquration $\varepsilon = 0.4$

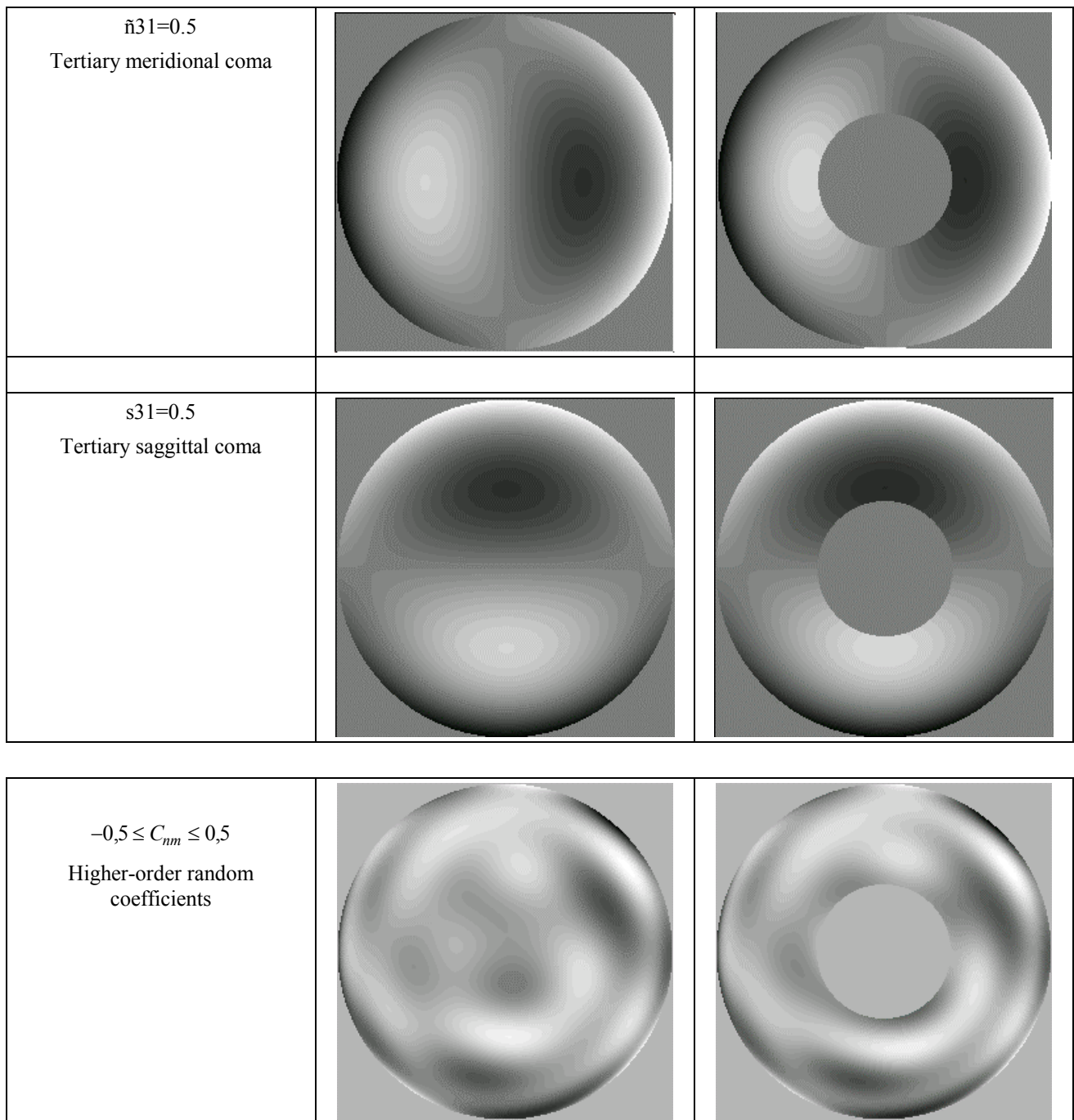|		
ñ31=0.5 Tertiary meridional coma		
s31=0.5 Tertiary saggittal coma		
$-0,5 \leq C_{nm} \leq 0,5$ Higher-order random coefficients		

Fig. 4. Topographical plots of mirror surface deformations for several aberration types and for random aberration coefficients

The deformation calculation accuracy is connected through a multiplier (-2) on the formula (9) to the wave front deformation coefficients calculation accuracy, formed by tested optical system.

The error of coefficients calculation is connected to an error of transverse aberrations measurement (the transverse aberrations dispersion = 0.05 mm), that follows from preliminary data processing. The decision of a problem brings in errors, determined by conditionality number, what, as was found out in result of numerous checks, much depends on Hartmann mask points distribution.

Best from the point of view of conditionality number there was the Hartmann mask with a radial arrangement of apertures, in this case the conditionality number is close to 1, in case of a rectangular grid it is close to 100.

CONCLUSION

In result of done work the mathematical apparatus, algorithms and software of astronomical optics Hartmann test final data processing with restoration of the deformed mirror form is developed.

Problems of orthogonal on ring area basis formation, matrix of the linear equations inversing process by means of the orthogonalization method, ensuring the highest reliability, were solved successfully. Also evident results of testing as reconstructed deformed surfaces are received.

The given technique is developed for to the operational control of BTA main mirror, but it is rather universal and can be used for final data processing of any optics control including objectives.

Due to good, close to ideal, method conditioning, the error of initial data does not increase at processing, that enables reliably to supervise accuracy of all mirror certification processes.

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